Chain rule with quotient rule

Chain rule is also often used with quotient rule. Let's look at an example of how these two derivative rules would be used together.

Example

Use chain rule to find the derivative.

$$y = \left(\frac{6x^4}{8\sin x}\right)^8$$

If we use substitution, then

$$u = \frac{6x^4}{8\sin x}$$

and according to quotient rule,

$$u' = \frac{\left(24x^3\right)(8\sin x) - \left(6x^4\right)(8\cos x)}{(8\sin x)^2}$$

Substituting into our original equation, we get

$$y = (u)^8$$

and using power rule with chain rule, the derivative is

$$y' = 8(u)^7(u')$$

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Back-substituting for u and u', the derivative is

$$y' = 8\left(\frac{6x^4}{8\sin x}\right)^7 \left[\frac{(24x^3)(8\sin x) - (6x^4)(8\cos x)}{(8\sin x)^2}\right]$$

$$y' = 8\left(\frac{3}{4}\right)^7 \frac{x^{28}}{\sin^7 x}\left[\frac{192x^3\sin x - 48x^4\cos x}{64\sin^2 x}\right]$$

$$y' = \left(\frac{3}{4}\right)^7 \frac{x^{28}}{\sin^7 x}\left[\frac{24x^3\sin x - 6x^4\cos x}{\sin^2 x}\right]$$

$$y' = \left(\frac{3}{4}\right)^7 \left[\frac{6x^{31}(4\sin x - x\cos x)}{\sin^9 x}\right]$$

Let's look at the combination of chain rule and quotient rule in a different way.

Example

Use chain rule to find the derivative.

$$y = \frac{(6x^4 - 5)^2}{(7x^2 + 3)^3}$$

In this case we want to do a double substitution, where

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$$u = 6x^4 - 5$$

$$u' = 24x^3$$

and

$$v = 7x^2 + 3$$

$$v' = 14x$$

Our original equation becomes

$$y = \frac{(u)^2}{(v)^3}$$

and applying power rule, quotient rule, and chain rule together, the derivative is

$$y' = \frac{(2u)(u')(v)^3 - (u)^2\left(3v^2\right)(v')}{\left[(v)^3\right]^2}$$

Back-substituting for u, u', v and v', we get

$$y' = \frac{2\left(6x^4 - 5\right)\left(24x^3\right)\left(7x^2 + 3\right)^3 - \left(6x^4 - 5\right)^2\left[3\left(7x^2 + 3\right)^2(14x)\right]}{\left[\left(7x^2 + 3\right)^3\right]^2}$$

$$y' = \frac{48x^3\left(6x^4 - 5\right)\left(7x^2 + 3\right)^3 - 42x\left(6x^4 - 5\right)^2\left(7x^2 + 3\right)^2}{\left(7x^2 + 3\right)^6}$$

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$$y' = \frac{6x\left(6x^4-5\right)\left[8x^2\left(7x^2+3\right)-7\left(6x^4-5\right)\right]}{\left(7x^2+3\right)^4}$$

$$y' = \frac{6x\left(6x^4-5\right)\left(56x^4+24x^2-42x^4+35\right)}{\left(7x^2+3\right)^4}$$

$$y' = \frac{6x\left(6x^4-5\right)\left(14x^4+24x^2+35\right)}{\left(7x^2+3\right)^4}$$

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